



# BIANCHI TYPE-V COSMOLOGICAL MODEL OF NON-LINEAR SPINOR FIELD COUPLING WITH ELECTROMAGNETIC FIELD

V R Patil<sup>1</sup>, N S Bayaskar<sup>2</sup>, G.U.Khapekar<sup>3</sup>, S.A.Bhojane<sup>4</sup>

<sup>1,2</sup>Department of mathematics, Arts, Science and Commerce College, Chikhaldara Dist. Amravati (M.S.), India

<sup>3</sup>Department of mathematics, Jagadamb College, Achalpur Dist. Amravati (M.S.), India

<sup>4</sup>Department of mathematics, P.R.Pote College of Engineering, Amravati Dist. Amravati (M.S.), India

## ABSTRACT

We have to consider an interacting system of nonlinear spinor and electromagnetic field within the scope of Bianchi type -V cosmological model. As a result, quadrature type solution is found. In the frame of the present cosmological model we have discussed the geometrical and physical properties of Bianchi type-V model.

**Key Words :** Bianchi type-V model, Nonlinear spinor field , Electromagnetic field.

## 1. INTRODUCTION

The discovery of the cosmic microwave radiation has motivated a rising interest in anisotropic general-relativistic cosmological models of the universe. The preference of anisotropic cosmological models in the system of Einstein field equations, the early day universe, which had an anisotropic phase that approaches an isotropic one. Misner [1] suggested that the dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation Bianchi type-I cosmological models that are anisotropic homogeneous universes play an important role in understanding essential features of the universe, such as formation of galaxies during its early stage of evolution. Thorne [2] has investigated locally Rotationally Symmetric (LRS) Bianchi type I model containing a magnetic field directed along one axis with a barotropic fluid. Jacobs [3, 4] investigated Bianchi type I models with magnetic field satisfying a barotropic equation of state. Bali [5] studied the behavior of the magnetic field in a

Bianchi type I universe for perfect fluid distribution. Bijan Saha et al [6] investigate Bianchi type VI model with cosmic strings in the presence of magnetic field. Also, B Saha [7] has studied Bianchi type I cosmological model filled with magneto fluid and he make attempt to study a system where all the four fluids, scalar, spinor, electromagnetic and gravitational ones play active part in the evolution process. Rybakov et al [8] has studied the system of spinor and electromagnetic field within the scope of Bianchi type I cosmological model and examine the influence of such interaction on expansion of the universe in the asymptotic region. Upadhaya et al [9] have investigated Bianchi type III massive string cosmological model in presence of magnetic field.

On the other hand, the magnetic field has an important role at the cosmological scale and is present in galactic and intergalactic spaces. Any theoretical study of cosmological models which contain a magnetic field must take into account that the corresponding universes are necessarily anisotropic. A large number of astrophysical observations prove the existence of magnetic fields in galaxies. Galactic magnetic fields which we observe today could be rest of a coherent magnetic field existing in the early Universe, before galaxy formation. Among the anisotropic space-time, Bianchi type-V space seems to be one of the most suitable for testing different cosmological models. The solutions of Einstein equations coupled to a spinor and a scalar field in Bianchi type I space-time have been extensively studied by Saha, Shikin and Sing [10-13]. Patil et al [14-17] obtained

cosmological models of thick domain walls with viscous field coupled with electromagnetic field in framework of Lyra geometry and has discussed the nature the existing model. Bali [18-19] has investigated Bianchi type-V string dust universe in the presence of magnetic field with variable magnetic permeability, to get the deterministic model, he assumed that  $F_{23}$  is the only non-vanishing component. Kandalkar et al [20] has investigated Bianchi type-V magnetized bulk viscous string dust cosmological model. It has been shown that the string dust cosmological model in the presence of bulk viscosity is not possible. Katore et al [21] investigate magnetized cosmological model. Pradhan et al [22] recommend Bianchi type-V universe with perfect fluid and heat flow in S'aez-Ballester scalar-tensor theory of gravitation by considering a law of variation of scale factor as increasing function of time which yields a time dependent deceleration parameter. Adhav et al [23] obtained vacuum Bianchi type -V string cosmological model with bulk viscous fluid. Singh, Ram and Zeyauddin [24] have extended the work of Singh and Kumar [25] to spatially homogeneous and totally anisotropic Bianchi type-V models with perfect fluid as source. The aim of this paper is to investigate Bianchi type -V cosmological model in presence of non linear spinor and electromagnetic field. Our paper is organized as follows: In section 2 we focus on material and gravitational field lagrangian. In Section 3 we have solve the field equations of Bianchi type -V cosmological model in the presence of a magnetic field and obtained the solution by using some simple conceivable assumptions and describe their asymptotic behavior , in section 4 we described some properties of model. The last Section 5 encloses conclusion.

**2. MATERIAL AND GRAVITATIONAL FIELD LAGRANGIAN**

Lagrangian density of spinor and gravitational field given by B Saha [26] in the form:

$$L_{sp} = \frac{R}{2\chi} + \frac{i}{2} [\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi] - m_{sp} \bar{\psi} \psi + F \tag{1}$$

with  $R$  being the scalar curvature,  $\chi$  being the Einstein's gravitational constant and  $m_{sp}$  is the spinor mass. The nonlinear term  $F$  described

the self-interaction of spinor field and it can be presented as some arbitrary function of invariants generated from the bilinear forms of spinor field.

$$S = \bar{\psi} \psi \tag{Scalar} \tag{2a}$$

$$P = i \bar{\psi} \gamma^5 \psi \tag{Pseudo scalar} \tag{2b}$$

$$v^\mu = \bar{\psi} \gamma^\mu \psi \tag{Vector} \tag{2c}$$

$$A^\mu = \bar{\psi} \gamma^5 \gamma^\mu \psi \tag{Pseudovector} \tag{2d}$$

$$Q^{\mu\nu} = \bar{\psi} \sigma^{\mu\nu} \psi \tag{Antisymmetric tensor}, \tag{2e}$$

where  $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu]$ .

The invariants corresponding to the bilinear forms are,

$$I = S^2 \tag{3a}$$

$$J = P^2 \tag{3b}$$

$$I_\nu = v_\mu v^\mu = (\bar{\psi} \gamma^\mu \psi) g_{\mu\nu} (\bar{\psi} \gamma^\mu \psi), \tag{3c}$$

$$I_A = A_\mu A^\mu = (\bar{\psi} \gamma^5 \gamma^\mu \psi) g_{\mu\nu} (\bar{\psi} \gamma^5 \gamma^\mu \psi), \tag{3d}$$

$$I_Q = Q_{\mu\nu} Q^{\mu\nu} = (\bar{\psi} \sigma^{\mu\nu} \psi) g_{\mu\alpha} (\bar{\psi} \sigma^{\alpha\beta} \psi). \tag{3e}$$

According to the Pauli-Fierz theorem [27] among the five invariants only  $I$  and  $J$  are independent as all other can be expressed by them:  $I_\nu = -I_A = I + J$  and  $I_Q = I - J$ . therefore, we choose the nonlinear term  $F = F(I, J)$ , thus claiming that it describes the nonlinearity in the most general of its form.[28]

**3. FIELD EQUATIONS**

We have consider the Bianchi-type-V metric in the form

$$ds^2 = dt^2 - a_1^2 e^{2mz} dx^2 - a_2^2 e^{2mz} dy^2 - a_3^2 dz^2 \tag{4}$$

in which  $a_1, a_2, a_3$  being the function of  $t$  only and  $m$  being some arbitrary constant.

The spinor field equation corresponding to the metric (1) has the form

$$i \gamma^\mu \nabla_\mu \psi - m_{sp} \psi + F_S \psi + i F_P \gamma^5 \psi = 0 \tag{5}$$

$$i \nabla_\mu \bar{\psi} \gamma^\mu + m_{sp} \bar{\psi} - F_S \bar{\psi} - i F_P \bar{\psi} \gamma^5 = 0 \tag{6}$$

where  $F_S = \frac{dF}{dS}$ ,  $F_P = \frac{dF}{dP}$  and  $\nabla_\mu$  is the covariant derivative of spinor field.

$$\begin{aligned} \nabla_\mu \psi &= \frac{\partial \psi}{\partial x^\mu} - \Gamma_\mu \psi \\ \nabla_\mu \bar{\psi} &= \frac{\partial \bar{\psi}}{\partial x^\mu} + \bar{\psi} \Gamma_\mu \end{aligned} \tag{7}$$

with  $\Gamma_\mu$  being the spinor affine connections for the metric (4) has the form

$$\Gamma_0 = 0 \tag{8}$$

$$\Gamma_1 = \frac{1}{2} \left( \dot{a}_1 \overline{\gamma^1 \gamma^0} + m \frac{a_1}{a_3} \overline{\gamma^1 \gamma^3} \right) e^{mz} \quad (9) \quad \frac{1}{2} \frac{\partial}{\partial t} (S^2 + P^2) + \frac{\dot{V}}{V} (S^2 + P^2) - 2(F_p S - F_s P) A^0 = 0 \quad (23)$$

$$\Gamma_2 = \frac{1}{2} \left( \dot{a}_2 \overline{\gamma^2 \gamma^0} + m \frac{a_2}{a_3} \overline{\gamma^2 \gamma^3} \right) e^{mz} \quad (10) \quad \text{As one sees, the assumption} \quad (24)$$

$$\Gamma_3 = \frac{1}{2} \dot{a}_3 \overline{\gamma^3 \gamma^0} \quad (11) \quad F_p S - F_s P = 0, \quad (24)$$

It implies that,  $F = F(S^2 + P^2) = C_0^2$  leads to  $V^2(S^2 + P^2) = C_0^2$  (25)

$$\Gamma_\mu \gamma^\mu = \frac{1}{2} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \overline{\gamma^0} + \frac{m}{a_3} \overline{\gamma^3} \quad (12) \quad \text{It can be easily verified that the relation (24) holds if one assume that,} \quad (26)$$

$$\gamma^\mu \Gamma_\mu = -\frac{1}{2} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \overline{\gamma^0} - \frac{m}{a_3} \overline{\gamma^3} \quad (13) \quad \text{The energy-momentum tensor of the system is given by} \quad (27)$$

Let us introduce a new function  $V = a_1 a_2 a_3$  (14)

$$T_{sp}^\nu = T_{sp\mu}^\nu + E_\mu^\nu \quad (27)$$

And taking into account that the spinor field is a function of  $t$  only, we obtained

$$\overline{\gamma^0} \left( \dot{\psi} + \frac{1}{2} \frac{\dot{V}}{V} \psi \right) + \frac{m}{a_3} \overline{\psi \gamma^3} + im_{sp} \overline{\psi} - iF_s \overline{\psi} + F_p \overline{\gamma^5 \psi} = 0 \quad (15)$$

$$\left( \dot{\overline{\psi}} + \frac{1}{2} \frac{\dot{V}}{V} \overline{\psi} \right) \overline{\gamma^0} + \frac{m}{a_3} \overline{\psi \gamma^3} - im_{sp} \overline{\psi} + iF_s \overline{\psi} - F_p \overline{\psi \gamma^5} = 0 \quad (16)$$

From (15) and (16) yields,  $\overline{\mu}$  is magnetic permeability and  $h_\mu$  is the magnetic flux vector define by

$$\dot{S} + \frac{\dot{V}}{V} S - 2F_p A^0 = 0 \quad (17) \quad E_\mu^\nu \text{ is electromagnetic field given by Lichnerowicz [29]} \quad (28)$$

$$\dot{P} + \frac{\dot{V}}{V} P - 2m_{sp} A^0 + 2F_s A^0 = 0 \quad (18) \quad E_\mu^\nu = \overline{\mu} \left[ |h|^2 \left( u_\mu u^\nu - \frac{1}{2} \delta_\mu^\nu \right) - h_\mu h^\nu \right] \quad (29)$$

$$\dot{A} + \frac{\dot{V}}{V} A^0 + 2mA^3 + 2m_{sp} P - 2F_s P + 2F_p S = 0 \quad (19) \quad \text{Here } u^\mu \text{ is the flow vector satisfying } g_{\mu\nu} u^\mu u^\nu = 1 \quad (30)$$

$$\text{Where } *F_{\mu\nu} \text{ is the dual electromagnetic field tensor as } h_\mu = \frac{1}{\mu} *F_{\nu\mu} u^\nu \quad (31)$$

$$*F_{\mu\nu} = \frac{\sqrt{-g}}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \quad (32) \quad \text{Here } F^{\alpha\beta} \text{ is the electromagnetic field tensor and } \epsilon_{\mu\nu\alpha\beta} \text{ is the totally anti-symmetric Levi-Civita tensor with } \epsilon_{0123} = +1$$

Equation (17), (18) and (19) with four unknown, to find the solution we have the condition  $A^3 = \alpha A^0$  (21)

From equations (17), (18) and (19) yields,  $V^2(S^2 + P^2 + A^{0^2}) e^{Amz} = Const.$  (22)

From equations (17) and (18) we have, Here the commoving coordinates are taken to consideration,  $u^0 = 1, u^1 = u^2 = u^3 = 0$  and  $u_\mu u^\mu = 1.$  (33)

We choose the magnetic field to be in the direction of  $x$ -axis, so that the magnetic flux vector has only one non-trivial component, viz  $h_1 \neq 0.$  In view of the above mentioned assumption, we have obtained  $F_{12} = F_{13} = 0$  and assume that the conductivity of the fluid is

infinite. This leads,  $F_{01} = F_{02} = F_{03} = 0$ . Thus, we have only one non-vanishing component of  $F_{\mu\nu}$  which is  $F_{23}$ . From the first set of Maxwell equation,

$$F_{\mu\nu;\beta} + F_{\nu\beta;\mu} + F_{\beta\mu;\nu} = 0 \quad (34)$$

where the semicolon stands for covariant derivative, we obtained

$$F_{23} = I, \text{ where } I = \text{constant} \quad (35)$$

Then from (31) in account of (35) we obtain

$$h_1 = \frac{a_1 I}{\mu a_2 a_3} \quad (36)$$

Finally we obtain the electromagnetic field tensor  $E_\mu^\nu$  as,

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (37)$$

Using equation (27), (28) and (37) the energy momentum tensors for given system are,

$$T_0^0 = m_{sp} S - F + \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz}, \quad (38a)$$

$$T_1^1 = k(SF_S + PF_P - F) + \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz}. \quad (38b)$$

$$T_2^2 = T_3^3 = k(SF_S + PF_P - F) - \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (38c)$$

$$T_3^0 = 0 \quad (38d)$$

Corresponding to the metric (4) and the energy momentum tensors in (38), the Einstein's field equations are obtained as;

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m^2}{a_3^2} = k(SF_S + PF_P - F)$$

$$+ \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (39)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_3^2} = k(SF_S + PF_P - F)$$

$$- \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (40)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_3^2} = k(SF_S + PF_P - F) - \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (41)$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{3m^2}{a_3^2} = k(m_{sp} S - F) + \frac{I^2}{2\mu a_2^2 a_3^2} e^{-2mz} \quad (42)$$

$$\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} - 2\frac{\dot{a}_3}{a_3} = 0 \quad (43)$$

Solving equation (43) we obtain,

$$a_1 a_2 = N a_3^2 \quad (44)$$

The expansion  $\theta$  is proportional to any of the components (say  $\sigma_1^1$ ) of the shear tensor  $\sigma$ , the condition was first used by Bali [5]. In this hypothesis we introduce the dynamical scalars such as the expansion and the shear scalar as usual.

$$\theta = u_{;\mu}^\mu, \sigma^2 = \frac{1}{2} \sigma^{\mu\nu} \sigma_{\mu\nu} \quad (45)$$

Where

$$\sigma_{\mu\nu} = \frac{1}{2} [u_{\mu;\alpha} P_\nu^\alpha + u_{\nu;\alpha} P_\mu^\alpha] - \frac{1}{3} \theta P_{\mu\nu} \quad (46)$$

Here  $P_{\mu\nu}$  is the projection tensor. For the space-time with signature (+, -, -, -) it has the form

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu, P_\nu^\mu = \delta_\nu^\mu - u^\mu u_\nu \quad (47)$$

For the Bianchi Type -V metric the dynamical scalar has the form

$$\theta = \Gamma_{\mu\nu}^\mu = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = \frac{\dot{V}}{V} \quad (48)$$

And  $\sigma_1^1 = \frac{\dot{a}_1}{a_1} - \frac{1}{3} \theta$  (49)

The proportionality condition

$$\sigma_1^1 = q_1 \theta, q_1 = \text{Const.} \quad (50)$$

It leads to,

$$a_1 = (a_2 a_3)^{\frac{1+3q_1}{2-3q_1}} \quad (51)$$

On account of equations (14), (44) and (51) we obtain,

$$a_1 = V^{\left(\frac{1+3q_1}{3}\right)} \quad (52)$$

$$a_2 = N^{\frac{1}{3}} V^{\left(\frac{1-3q_1}{3}\right)} \quad (53)$$

$$a_3 = \left(\frac{V}{N}\right)^{\frac{1}{3}} \quad (54)$$

Adding the equations (39), (40), (41) and 3 times (42) and using (52),(53),(54) we obtain,

$$\frac{2\ddot{V}}{V} = \left[ 12m^2 N^{\frac{2}{3}} V^{\frac{-2}{3}} \right] + \frac{I^2}{\mu} V^{\frac{-4+6q}{3}} e^{-2mz} + 3k\{m_{sp}S + (SF_S + PF_P - 2F)\} \quad (55)$$

The equation (55), can be written as,

$$2\ddot{V} = 12m^2 N^{\frac{2}{3}} V^{\frac{1}{3}} + \frac{I^2}{\mu} V^{\frac{-1+6q}{3}} e^{-2mz} + 3k\{m_{sp}S + (SF_S + PF_P - 2F)\}V \quad (56)$$

At last we assume that the spinor field be a massless one and the spinor field nonlinearity is given by  $F = F(K)$  with  $K = S^2 + P^2$ .

In this case  $F_S = 2SF_K$  and  $F_P = 2PF_K$ , hence  $SF_S + PF_P = 2(S^2 + P^2)F_K = 2KF_K$ .

The equation (56) gives,

$$2\ddot{V} = 12m^2 N^{\frac{2}{3}} V^{\frac{1}{3}} + \frac{I^2}{\mu} V^{\frac{-1+6q}{3}} e^{-2mz} + 6k(kF_K - F)V \quad (57)$$

Let us choose the spinor field nonlinearity in the concrete form  $F = K^n$ , also taking into

account that  $K = S^2 + P^2 = \frac{C_0^2}{V^2}$  we rewrite

(57) as;

$$\ddot{V} = 6m^2 N^{\frac{2}{3}} V^{\frac{1}{3}} + \frac{I^2}{2\mu} V^{\frac{-1+6q}{3}} e^{-2mz} + 3k(n-1)C_0^{2n} V^{(1-2n)} \quad (58)$$

$$\dot{V}^2 = 9m^2 N^{\frac{2}{3}} V^{\frac{4}{3}} + \frac{3I^2}{2\mu(1+3q)} V^{2\left(\frac{1+3q}{3}\right)} e^{-2mz} - 3kC_0^{2n} V \quad (59)$$

we obtain the solution in quadrature

$$\int \frac{dV}{\sqrt{AV^{\frac{4}{3}} + BV^{2\left(\frac{1+3q}{3}\right)} e^{-2mz} - 3kC_0^{2n} V^{2(1-n)} + C_1}} = t + t_0 \quad (60)$$

Where  $A = 9m^2 N^{\frac{2}{3}}$ ,  $B = \frac{3I^2}{2\mu(1+3q)}$

In which  $C_1, t_0$  being some integration constant. In unkindness of that this equation cannot be explicitly solved, the asymptotic behavior of the solution for  $t \rightarrow \infty$  could be found.

It should be mentioned that being the volume-scale  $V$  is non-negative. At the points when  $V = 0$  the space-time occurs singularity.

#### 4. SOME PROPERTIES OF THE MODEL

The Non vanishing component of shear tensor  $\sigma_{\mu\nu}$  are given by,

$$\sigma_{11} = \left[ -a_1 \dot{a}_1 + \frac{a_1^2}{3} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \right] e^{2mz} \quad (61a)$$

$$\sigma_{22} = \left[ -a_2 \dot{a}_2 + \frac{a_2^2}{3} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \right] e^{2mz} \quad (61b)$$

$$\sigma_{33} = \left[ -a_3 \dot{a}_3 + \frac{a_3^2}{3} \left( \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \right] e^{2mz} \quad (61c)$$

$$\sigma_{44} = 0 \quad (61d)$$

Therefore,

$$2\sigma^2 = \frac{\dot{a}_1^2}{a_1^2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{\dot{a}_3^2}{a_3^2} - \frac{1}{3}\theta \quad (62)$$

The model, in general represent shearing universe.

#### 5. CONCLUSIONS

We have studied Bianchi type-V cosmological model in presence of non linear spinor field and magnetic flux and obtained the solution in quadrature form (60). Also from equation (62)

we have observed that, the  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$

the model dose not tend to isotropic for large  $t$ . Further, we have studied asymptotic behavior of equation (58). It is clearly observed that, all the

physical quantities were constructed from the spinor field as well as the invariants of gravitational field are inverse function of  $V$ . It can be conclude that at any space time point where the volume scale becomes zero, it is singular point.

REFERENCES

- [1] C.W.Misner, *Astrophys. J.* 151, 431 (1968).  
 [2] K.S.Thorne, *Astrophys. J.* 148, 51 (1967).  
 [3] K.C.Jacobs, *Astrophys. J.* 153, 661(1968).  
 [4] K.C.Jacobs, *Astrophys. J.* 155, 379(1969).  
 [5]R.Bali, *Int. J. Theor. Phys.* 25, 755(1986).  
 [6] B.Saha, Mihai Visinescu, *Rom. Journ Phys.* 55, 1064. (2010).  
 [7]B.Saha,*Astrophysics and Space-science* 299, 149.(2005).  
 [8] Yu.P.Rybakov. G.N.Shikin. Yu.A.Popov B.Saha, *arXiv*, 1008. 2113v1[gr-qc](2010).  
 [9] R.D Upadhaya,S.Dave, *Brazilian Journal of Physics* 38(4), 615 (2008).  
 [10]B.Saha, G.N.Shikin, *Gen. Relat. Gravitat.* 29, 1099 (1997).  
 [11]B.Saha, G.N.Shikin , *J. Math. Phys.* 38, 5305(1997).  
 [12]B.Saha, *Modern Phys. Lett. A* 16, 1287. (2001b).  
 [13]A.Singh,R.C.Upadhyay,A.Pradhan, *ARPN Journal of Science and Technology*.3(2) ,146 (2013).  
 [14] V.R.Patil, S.A.Bhojane ,N.S.Bayaskar, *Vidarbha Journal of Science* 9(2) , 92-96(2014).  
 [15] V.R.Patil,P.A.Bolke , N.S.BayaskarN.S,*Int J.Theor.Phys.* DOI 10.1007/s10773-014-2175 -9 ( 2014).  
 [16]V.R.Patil , D.D.Pawar ,G.U. Khapekar , *Int.J.Theor.Phys*,DOI 10.1007/s10773- 012, 1088 -8(2012).  
 [17]V.R.Patil, D.D.Pawar, A.G.Deshmukh , *Romanian Reports in Physics* 62(4)722-30(2010).  
 [18]Raj Bali, *Electronic Journal of Theoretical Physics*, 5( 19) ,105-114(2013).  
 [19] Raj Bali, Anjali, *Pramana Journal of Physics*,63( 3),481-490(2004).  
 [20]S.P.Kandalkar,A.P.Wasnik,S.P.Gawande, *International Journal of Scientific & Engineering Research* 4(3),1-4(2013).  
 [21] S.D.Katore, R.S.Rane, *Pramana Journal of Physics* , 67( 2) (2006).  
 [22]A. Pradhan, A. Singh ,D. S. Chouhan, *Palestine Journal of Mathematics*,3(1) ,130-140(2014).  
 [23]K.S.Adhav,A.S. Nimkar, M.V.Dawande ,M.R.Ugale, *Rom.Journ.Phys.*,54(1),207( 2009)  
 [24] C. P Singh,, S.Ram, M .Zeyauddin, *Astrophys. Space Sci.*, 315, 181 (2008).  
 [25] C. P .Singh, S .Kumar, *Int. J. Mod. Phys. D*, 15, 419 (2006).  
 [26]B.Saha , *The Open Nuclear & Particle Physics Journal*,4,1-3(2011).  
 [27] M.O. Ribas , F.P.Devecchi, G.M. Kremer, *Physical Review D* 72 (12),123502 (2005).  
 [28]B. Saha, *Phys. Rev. D* 64, 123501(2001a)  
 [29]A.Lichnerowicz, *Relativistic Hydrodynamics and Magneto Hydrodynamics*, Benjamin, New York,13(1967).