ISSN 2277 - 5730

AN INTERNATIONAL MULTIDISCIPLINARY QUARTERLY RESEARCH JOURNAL

AJANTA

Volume - VIII Issue - I Part - VIII January - March - 2019

Peer Reviewed Refereed and UGC Listed Journal

Journal No. 40776

IMPACT FACTOR / INDEXING

2018 - 5.5

www.sjifactor.com

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Ajanta Prakashan

Aurangabad. (M.S.)

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2. A Bianchi Type-V Cosmological Models with Stiff Fluid in Theory of Gravity

V. R. Patil
Department of Mathematics, Arts, Commerce and Science College, Chikhaldara, Dist. Amravati, India.
D. D. Pawar
School of Mathematical Sciences, Swami Ramanand Teerth Marathwada University, Nanded, India.
R. V. Mapari
Department of Mathematics, Government Vidarbha Institute of Science and Humanities, Amravati, India.
J. L. Pawade
Department of Mathematics, P.R.PoteCollege of Engineering and Management, Amravati, India.

Abstract

In this paper, we investigate the Bianchi type-V cosmological model in a modified theory of gravitation proposed by Harko et al. (2011) when the source for energy-momentum tensor is stiff fluid. To solve the field equations we used the equation of state for the pressure and density and pressure is assumed to be proportional to energy density i.e. $\rho = p$. Some physical properties of the model are also discussed.

Keywords: Bianchi type –V space-time, $f(R,T)$ gravity, Energy momentum tensor for stiff fluid.

1. Introduction

It is well known that the late time accelerated expansion of the universe has been confirmed by the high red-shift supernovae experiments (Reiss et al. 1998; Perlmutter et. al. 1999, Bennett et al. 2003) and by the observations such as cosmic microwave background radiation (Spergel et al. 2007). In view of this, it is now believed that the energy composition of the universe has 4 % ordinary matter and 20 % dark matter and 76 % dark energy. The modifications of Einstein's theory are attracting more and more attention, in recent years, to explain the late time acceleration and dark energy. Among the various modifications of Einstein's theory, $f(R)$gravity (Akbar and Cai2006) and $f(R, T)$ gravity (Harko et al. 2011) theories are attracting more and more attention during the last decade because these theories are supposed to provide natural gravitational alternatives to dark energy. It has been suggested that the cosmic acceleration can be achieved by replacing Einstein-Hilbert action of general relativity with a general function $f(R)$, where R is a Ricci scalar. Chiba et al. (2007), Nojiri and Odintsov

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(2007, 2010), Multamaki and Vilja (2006, 2007) are some of the authors who have investigated several aspects of *f (R)* gravity models which show early time inflation and late time acceleration. A comprehensive review on *f (R)* gravity is given by Copeland et al. (2006). Another modification of standard general relativity is *f (R, T)* gravity proposed by Harko et al (2011) wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar *R* and of the trace of the stress energy tensor *T*. The gravitational field equations have been derived from the Hilbert-Einstein type variational principle modifications of Einstein's theory are attracting more and more attention, in recent years, to explain the late time acceleration and dark energy. Dark energy cosmological models in alternative theories of gravitation has been investigated by Pawar & Solanke 2014, Samanta 2013 and Pawar et. al. 2014. On the basis of this data, in the present study, Bianchi type-V space-time is taken into consideration. Now a day's f(R, T) theory has attracted a lot of attention for the astrophysicist in recent times because of its ability to explain a lot of issues in cosmology and astrophysics (Barrow, J.D., Turner, M.S. 1981, Sahoo P. K., et. al. 2016, Yousaf Z., Ilyas M., Bhatti M. Z., 2017). Pawar et. al. 2017 studied the f(R, T) theory in Bianchi Type V model, Pawar et. al. 2016 also have analyzed two fluid axially symmetric cosmological models in f(R, T) theory of gravitation. Pawar et. al. 2017, have studied Bianchi type-V universe model with magnetized domain walls in f(R, T) theory. Also, Pawar and Agrawal 2017 obtained a plane-symmetric cosmological model with quark and strange quark matter in f (R, T) theory of gravity. In a recent year Pawar and Mapari 2019, studied a modified holographic ricci dark energy model in f(R,T) theory of gravity.

Mostly the expansion of the universe is described within the framework of the homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmology. The reasons for this are purely technical. The simplicity of the field equations and the existence of analytical solutions in most of the cases have justified this over simplification for the geometry of space-time. However, there are no compelling physical reasons to assume the former before the inflationary period. To drop the assumption of homogeneity would make the problem intractable, while the isotropy of the space is something that can be relaxed and leads to anisotropy. Several authors (Jaffe, J., et al., 2005, Jaffe, J., et al. 2006, Campanelli, L., Cea, P., 2006, Campanelli, L., Cea, P., 2007) have studied particular cases of anisotropic models and found that the scenario predicted by the FRW model stand essentially unchanged even when large anisotropies were present before the inflationary period. Thus, the anisotropic Bianchi models becomes interesting in academics. For more details one can refer research papers from various authors [27-32]

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The f(R, T) theory of gravity is the generalization or modification of General Relativity (GR). In this theory, the modified gravity action is given by

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \quad (1)$$

where $T = g^{ij}T_{ij}$, trace T of stress energy tensor of the matter T_{ij}. $f(R,T)$is an arbitrary function of the Ricci scalar R and $L_m$represents matter Lagrangian density depending only on metric tensor components g_{ij}. Also, the trace of tress energy tensor of matter is defined as:

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}} \quad (2)$$

Assuming that the Lagrangian density $L_m$of the matter is depend only on the components of methe tric tensor g_{ij} and not on derivatives, we have

$$T_{ij} = g_{ij}L_m - \frac{2\partial L_m}{\partial g^{ij}} \quad (3)$$

Now by varying action S of the gravitational field with respect to the metric tensor components g_{ij}we obtain the field equations of $f(R,T)$gravity as,

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \Delta_i\Delta_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad (4)$$

$$\text{Where}\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\delta^2 L_m}{\delta g^{ij}\delta g^{lk}} \quad (5)$$

Here, $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$,$\Box = \Delta^i\Delta_i$ and $\Delta_i$is the covariant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m. It may be noted that when $f(R,T) \equiv f(R)$ the equation (4) yields the field equation of $f(R)$ gravity.

The problem of stiff fluids described by energy density, pressure and four velocity vector $u^i$is complicated since there is no unique definition of matter Lagrangian and we have the four velocity in the moving coordinates that satisfies the conditions,

$$u^i u_j = 1 \text{And} u^i\nabla_j u_i = 0 \quad (6)$$

where $u^i = (0,0,0,1)$.

From equation (5) we have obtained the variation of stress-energy for perfect fluid is,

$$\theta_{ij} = -2T_{ij} - pg_{ij} \quad (7)$$

Generally, in physics, the nature of matter field equations are also depended through tensor θ_{ij}.In case of $f(R,T)$gravity it depending on the nature of source matter, we assume that,

$$f(R,T) = f_1(R) + f_2(T) \quad (8)$$

Where $f_1(R)$ is a function of theof Ricci scalar R and $f_2(T)$is a function of the trace of stress energy tensor of matter$T$.

Using equation (9) in equation (4) we have obtained the gravitational field equations of $f(R,T)$ gravity as,

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + (g_{ij}\Box - \Delta_i\Delta_j)f_1'(R) = 8\pi T_{ij} - f_2'(T)T_{ij} + \left[f_2'(T)p + \frac{1}{2}f_2(T)\right]g_{ij} \quad (9)$$

Take $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$

Equation (10) takes the form,

$$\lambda R_{ij} - \frac{1}{2}\lambda(R+T)g_{ij} + (g_{ij}\Box - \Delta_i\Delta_j)\lambda = 8\pi T_{ij} - \lambda T_{ij} + \lambda(2T_{ij} + pg_{ij}) \quad (10)$$

Setting $(g_{ij}\Box - \Delta_i\Delta_j) = 0$, we get

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{ij} + \left(p + \frac{1}{2}T\right)g_{ij}$$

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{ij} + \Lambda g_{ij} \quad (11)$$

Where $\Lambda = \left(p + \frac{1}{2}T\right)$ termed as a cosmological constant. (12)

2. Metric and Field Equations

We have considered the general class of Bianchi type –V cosmological space-time described by the line element as,

$$ds^2 = dt^2 - A(t)^2dx^2 - e^{2\beta x}[B(t)^2dy^2 + C(t)^2dz^2] \quad (13)$$

where A, B and C are the three anisotropic directions of expansion in normal three dimensional space and are the functions of cosmic time t and β is a constant

The energy-momentum tensor for a stiff fluid is,

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} \quad (14)$$

The spatial volume (V) and the average Hubble parameter (H)are defined as

$$V = ABC \quad (15)$$

$$H = \frac{1}{3}\left(\frac{H_1}{H} + \frac{H_2}{H} + \frac{H_3}{H}\right) \quad (16)$$

Here $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameter in the direction of x,y and z axes respectively.

The physical quantities of observational interest in cosmology are expansion scalar θ, deceleration parameter q, shear scalar σ^2 and the mean anisotropic parameter A_m which are defined as,

$$\theta = 3H = 3\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (17)$$

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \quad (18)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad (19)$$

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$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad (20)$$

Using energy-momentum tensor (14), for the metric (13) the field equation (11)yields,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\beta^2}{A^2} = -\left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \quad (21)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\beta^2}{A^2} = -\left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \quad (22)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\beta^2}{A^2} = -\left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \quad (23)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\beta^2}{A^2} = -\left(\frac{8\pi + \lambda}{\lambda}\right)(2p + \rho) - \Lambda \quad (24)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (25)$$

in which the overhead dot denotes the derivative with respect to t.

Integrating equation (25) and absorbing the constant of integration, we have

$$A^2 = BC \quad (26)$$

Now equations (21)–(25) are a system of four independent equations in six unknowns, A, B, C, p, ρ, and Λ. Also, the equations are highly nonlinear. Hence to find a determinate solution we use the following physically plausible condition, stiff fluid cosmological models have generated interest because in these fluids the speed of light is equal to the speed of sound and the governing equations have the same characteristics as that of the gravitational field. The equation of state of the stiff fluid is given as

$$\rho = p \quad (27)$$

Now from equations (21), (22) and (23), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C} = a \quad (28)$$

on integrating we have,

$$A(t) = B(t) = C(t) = e^{at+k} \quad (29)$$

where k is the constant of integration.

Using equation (29), the metric (13) takes the form as,

$$ds^2 = dt^2 - e^{2(at+k)}\left[dx^2 + e^{2\beta x}(dy^2 + dz^2)\right] \quad (30)$$

3. Some physical properties

Equation (30) represents the Bianchi type –V stiff fluid cosmological model in *f (R, T)* modified theory of gravitation with the following physical kinematical parameters which play an important role in the discussion of cosmology

The spatial volume

$$V = e^{3(at+k)} \quad (31)$$

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The Hubble parameter

$$H = a \tag{32}$$

Scalar of expansion

$$\theta = 3H = 3a \tag{33}$$

The deceleration parameter

$$q = -1 \tag{34}$$

The shear scalar

$$\sigma^2 = \frac{3}{2}a^2 \tag{35}$$

The mean anisotropic parameter

$$A_m = 0 \tag{36}$$

Using equation (27) and equation (29) in equation (23) - (24) and further solving, we obtain the pressure (p), density (ρ) and the cosmological constant (Λ) for the model (30) as,

$$p = \rho = \left(\frac{\lambda}{8\pi+\lambda}\right)\frac{\beta^2}{e^{2(at+k)}} \tag{37}$$

$$\Lambda = 8\left(\frac{\lambda}{8\pi+\lambda}\right)\frac{\beta^2}{e^{2(at+k)}} \tag{38}$$

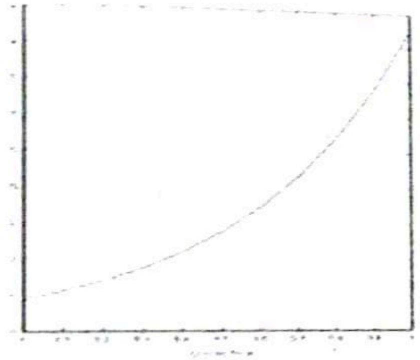

Fig.1 Spatial Volume Vs Cosmic Time

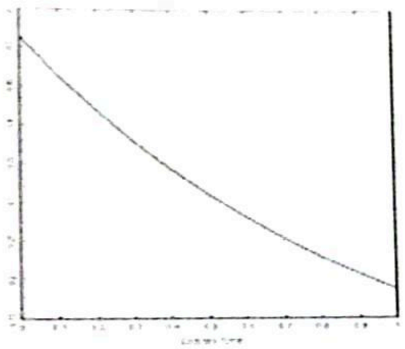

Fig.2 Pressure Vs Cosmic Time

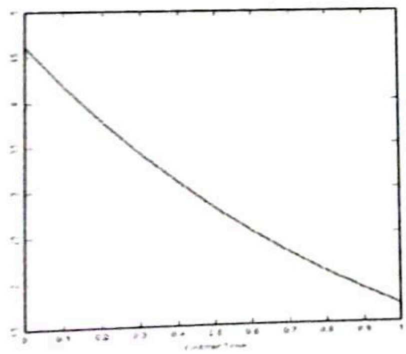

Fig.3 Cosmological Constant Vs Cosmic Time

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In above figures all parameters are in arbitrary units.

4. Conclusion

In this paper, we have investigated the Bianchi type –V cosmological model in $f(R,T)$ theory of gravity with stiff fluid. From equation (37) and (38) it is clear that the pressure, density and cosmological constant are inversely proportional to cosmic time i.e as time increases pressure, density and cosmological constant decreases gradually. Also as $t \to \infty$, pressure, density and cosmological constant approaches to zero, it leads to inflationary cosmological model and has initial singularity as $t \to 0$. Also, we found that the deceleration parameter $q = -1$, that means the universe is accelerating with a constant rate. Anisotropy parameter is equal to zero shows that our model approaches to isotopy.

Also from Fig.1, it is clear that as cosmic time t increases the spatial volume goes on increasing and hence universe is expanding.

Fig.2 Shows that the pressure decreases as cosmic time increases.

Fig.3 shows that as cosmic time increases the value of cosmological constant is decreases.

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